TEACHING PLAN: Special Functions and Integral Transforms

SCHOOL: (SOBAS) SCHOOL OF BASIC AND APPLIED SCIENCE		**ACADEMIC SESSION: 2023 - 2024**	**FOR STUDENTS' BATCH: B.Sc 2nd Sem.**
1	**Course No.**	MA-204	
2	**Course Title**	**Special Functions and Integral Transforms**	
3	**Credits**	**2**	
4	**Learning Hours**	Per week two lectures Total hours; 28	
5	**Course Objective**	1. Master Series Solutions: Understand power series method, Beta and Gamma functions, Bessel equation, and Legendre and Hermite differential equations. 2. Grasp Laplace Transforms: Learn Laplace transform properties, existence theorem, and application in solving ordinary differential equations. 3. Explore Fourier Transforms: Study finite and infinite Fourier transforms, Fourier integral, and their applications in boundary value problems. 4. Apply Transform Theorems: Utilize shifting, convolution, and differentiation theorems for Laplace and inverse Laplace transforms.	
6	**Course Outcomes**	1. Series Solutions Proficiency: Apply power series and Bessel functions to solve differential equations, grasp properties of Legendre and Hermite functions. 2. Laplace Transform Mastery: Demonstrate understanding of Laplace transform properties, effectively solve differential equations, and apply convolution theorem. 3. Fourier Transform Application: Utilize Fourier transforms to solve boundary value problems, understand finite and infinite transforms, and apply Fourier integral. 4. Transform Theorem Application: Apply Laplace and inverse Laplace transforms with shifting and convolution theorems to solve differential equations effectively.	
7	**Outline syllabus:**		

7.01	**Paper Code**	**Unit**	**Introduction**	**Page Numbers[1]**	**Lectures**
7.02	**Unit I**	(a)	Series solution of differential equations -Power series method, Definitions of Beta and Gamma functions.	1.1 to 1.11	1,2
		(b)	Bessel equation and its solution: Bessel functions and their properties-Convergence, recurrence,	2.1 to 2.15	3,4
		(c)	Relations and generating functions, Orthogonality of Bessel functions.	3.1 to 3.16	5,6,7
7.03	**Unit II**	(a)	Legendre and Hermite differentials equations and their solutions: Legendre and Hermite functions	**3.17 to 3.30**	**8,9,10**
		(b)	and their properties-Recurrence Relations and generating functions. Orhogonality of Legendre and	**3.31 to 3.35**	**11,12**

		(c)	Hermite polynomials. Rodrigues' Formula for Legendre & Hermite Polynomials, Laplace Integral	**4.1 to 4.50**	**13,14**
7.04	**Unit III**	(a)	Laplace Transforms – Existence theorem for Laplace transforms, Linearity of the Laplace transforms,	**5.1 to 5.13**	**15,16**
		(b)	Shifting theorems, Laplace transforms of derivatives and integrals, Differentiation and integration of	**6.1 to 6.25**	**17,18, 19**
		(c	Laplace transforms, Convolution theorem.	**6.26 to 6.30**	**20,21**
7.05	**MA 303 Unit IV**	(a)	Inverse Laplace transforms, convolution theorem, Inverse Laplace transforms of derivatives and integrals	**7.1 to 7.17**	**22,23**
		(b)	change the scale property and Shifting theorems for inverse Laplace	**7.18 to 7.30**	**24,25, 26**
		(c)	solution of ordinary differential equations using Laplace transform. Finite Fourier transforms, Infinite Fourier transforms,	**8.1 to 8.13**	**27,28**

8	**Course Evaluation**	
8.1	**Attendance**	5%
8.2	**Homework**	4 Assignments, 5%
8.3	**Quizzes**	2Quizzes, 5%
8.4	**Projects**	1 Project, 5%
8.5	**Presentation**	1 Presentation, 5%
8.2	**MTE**	20%
8.3	**End-term examination: 60%**	
9	**Text Books & References**	
9.1	**Text book**	1. I.N. Sneddon: the use of integral transform, McGraw Hill, 1972 2. Murray R. Spiegel: Laplace transforms, Schaum's Series. 3. S. S Seth Integral Transforms : Students' Friends & Company 4. I.N. Sneddon: Special Functions on mathematics, Physics & Chemistry. 5. W.W. Bell: Special Functions for Scientists & Engineers.
9.2	**References**	
9.3	**Video References**	

[Q.1]

a) Express the following in term of x^k and verify the result

$\sum_{m=1}^{\infty} m a_m x^{m-1}$

. **b)** Show that x= 0 is an ordinary point of the differential equation

$(x^2-1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - xy = 0$

c) Define the relation between Beta and Gamma Function

d) Define the Laplace of following term (i) L sinh5t, (ii) t^n

e) Find the value of $J_1(x)$, $J_2(x)$.

f) Find the value of $P_0(x)$, $P_1(x)$.

[Q.2]

Integrate the series the differential equation

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0 \text{ (in the power series)}$$

[Q.3]

a) To show that $\frac{d}{dx}\left(x^n J_{n(x)}\right) = x^n J_{n-1(x)}$

b) Show that J4 (x) $= \left(\frac{48}{x^3} - \frac{8}{x}\right) J_1(x) + \left(1 - \frac{24}{x^2}\right) J_{(0)}(x)$

[Q.4]

a) Verify the Legendre polynomials $p_3(x) = \frac{1}{2}(5x^3-3x)$ satisfies the Legendre equation when the parameter n is equal to 3.

b) Express $x^4 + 2x^3 + 2x^2 - 3$ in term of Legendre's polynomials

[Q.5]

a) Express $H_5(x) = 32x^5 - 160x^3 + 120x$ in terms of the Hermitan polynomials.

b) Find the value of H_1, H_2, H_3.

[Q.6]

a) Find the Laplace transform of the functions $\sinh 3t \cos^2 t$.

b) Apply the Convolution theorem to evaluate $L^{-1}\frac{s}{(s^2+a^2)^2}$.

[Q.7]

a) Find the inverse Laplace transform of $\frac{2s^2 - 6s+5}{s^3-6s^2+11s-6}$.

b) Find the inverse Laplace transform of $\frac{s}{(s^2+a^2)^2}$.

[Q.8]

Solve the following equation by transform method

$\frac{d^2y}{dx^2}$ + 6y = 6cos2t, where $y'(0) = 1$, y (0) =3.

[Q.9] Find the Fourier transform of f(x) = $e^{-a|x|}$ where a >0 and x ∈ (-∞, ∞)