TEACHING PLAN: LINEAR ALGEBRA

SCHOOL: (SOBAS) SCHOOL OF BASIC AND APPLIED SCIENCE		ACADEMIC SESSION: 2023 – 2024	FOR STUDENTS' BATCH: B.Sc VIth Sem.
1	**Course No.**	MA 303	
2	**Course Title**	**LINEAR ALGEBRA**	
3	**Credits**	**2**	
4	**Learning Hours**	Per week two lectures Total hours; 28	
5	**Course Objective**	1. Explain the fundamental concepts of advanced algebra such as groups and rings and their role in modern mathematics and applied contexts 2. Accurate and efficient use of advanced algebraic techniques 3. Demonstrate capacity for mathematical reasoning through analyzing, proving and explaining concepts from advanced algebra 4. Apply problem-solving using advanced algebraic techniques applied to diverse situations in physics, engineering and other mathematical contexts	
6	**Course Outcomes**	After completing the course, the students will be able to:The importance of group in algebra. 1. The importance of a ring as a fundamental object in algebra. 2. The importance of group in algebra. 3. The concept of a module as a generalization of a vector space and an Abelian group. 4. Homomorphism and isomorphism, Cayley's theorem, Normal subgroups, Quotient group. 5. The concept of Sub ring Homomorphism and isomorphism, field, Field of quotients of an integral domain	
7	**Outline syllabus:**		

7.01	**Paper Code**	**Unit**	**Introduction**	**Page Numbers[1]**	**Lectures**
7.02	**Paper Code. MA 303 Unit I**	(a)	Definition and properties of vector spaces.	1.1 to 1.11	1,2
		(b)	Subspaces: Definition, properties, and examples.	2.1 to 2.15	3,4
		(c)	Operations: Addition, scalar multiplication, and linear combinations.	3.1 to 3.16	5,6,7
7.03	**Paper Code. MA 303 Unit II**	(a)	Linear transformations: Definitions and properties.	**3.17 to 3.30**	**8,9,10**
		(b)	Isomorphisms of vector spaces: Definitions and significance.	**3.31 to 3.35**	**11,12**
		(c)	Homomorphisms: Understanding their role in preserving structure.	**4.1 to 4.50**	**13,14**
7.04	**Paper Code. MA 303 Unit III**	(a)	Homomorphisms between vector spaces.	**5.1 to 5.13**	**15,16**
		(b)	Isomorphisms and preservation of structure.	**6.1 to 6.25**	**17,18,**

					19
		(c	Examples illustrating homomorphisms and isomorphisms.	**6.26 to 6.30**	**20,21**
7.05	**Paper Code. MA 303 Unit IV**	(a)	Inner product spaces: Definitions and properties.	**7.1 to 7.17**	**22,23**
		(b)	Orthogonal vectors and sets: Concepts and applications.	**7.18 to 7.30**	**24,25, 26**
		(c)	Orthogonal complements: Understanding their significance in vector spaces.	**8.1 to 8.13**	**27,28**

8	**Course Evaluation**	
8.1	**Attendance**	5%
8.2	**Homework**	4 Assignments, 5%
8.3	**Quizzes**	2Quizzes, 5%
8.4	**Projects**	1 Project, 5%
8.5	**Presentation**	1 Presentation, 5%
8.2	**MTE**	20%
8.3	**End-term examination: 60%**	
9	**Text Books & References**	
9.1	**Text book**	1. 1. Linear Algebra, Jeevansons publication 2. I.N. Herstein : Topics in Algebra, Wiley Eastern Ltd., New Delhi
9.2	**References**	1. P.B. Bhattacharya, S.K. Jain and S.R. Nagpal : Basic Abstract Algebra (2nd edition) 2. VivekSahai and VikasBist : Algebra, NKarosa Publishing House. 3. J.B. Gallian: Abstract Algebra, Narosa Publishing House.
9.3	**Video References**	1. https://youtu.be/2aLV4N7vd8U 2. mathonline.wikidot.com/...structures-fields-rings-and-groups 3. users.metu.edu.tr/matmah/Graduate-Algebra-Solution 4. www.ring-group.com

a. Define the subspace.
b. Explain the linear combination of the vector.
c. What is the linear transformation or vector space homomorphism?
d. Define the null space of a linear transformation.
e. Define the bi-linear mapping in vector space.
f. Define the norm of a vector space.

[Q.1]

a) A non-empty subset W of a vector space V(F) is a subspace of V if and only if au+bv € W for a,b € F and u,v € W.

b) The intersection of two subspaces W_1 and W_2 of a vector space V(F) is also a subspace of V(F).

[Q.2]

a) Express the vector v =(1, -2, 5) as linear combination of the vectors V_1 =(1,1,1), V_2 =(1,2,3), V_3 =(2,-1,-1) in the vector space R^3 (R).

b) Express the matrix $\begin{bmatrix} 2 & 0 \\ 4 & -5 \end{bmatrix}$ as a linear combination of the matrices A=$\begin{bmatrix} 0 & -3 \\ 2 & 0 \end{bmatrix}$, B=$\begin{bmatrix} 0 & 0 \\ 2 & 1 \end{bmatrix}$ and C=$\begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}$.

[Q.3]

a) Show that the function T: $R^3 \to R$ defined by T(x_1, x_2, x_3) = $x_1^2+x_2^2+x_3^2$ is not a linear transformation.

b) Show that the map T: $R^3 \to R^2$ defined by T(x, y, z) = (|x|, y, -z) is not a linear transformation.

[Q.4]

State and proof the fundamental theorem of homomorphism.

[Q.5]

a) Let T_1 : U→ V and T_2 : V→W be two linear transformations. Then

i. If T_2 T_1 is one-one, then T_1 is one-one.

ii. If T_2 T_1 is onto, then T_2 is onto.

b) Show that the linear transformation T : $R^2 \to R^3$ defined by

T(x, y) = (x+y, x-y, y) is a non- singular transformation.

[Q.6]

a) Find the eigen values and the corresponding eigen vectors of matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

a) If ϕ : V×V→W is bilinear, then

ϕ(x+y, x+y)-ϕ(x-y, x-y) + i ϕ(x+ iy, x+iy) – i ϕ(x- iy,x- iy)=0

b) If A is any normed algebra, then the bilinear mapping ϕ: A×A→A defined by ϕ(a, b)=a b is bounded.

[Q.7]

State and prove the Cauchy Schwarz inequality theorem.