



**TEACHING PLAN: Engineering Mathematics-II**

<b>SCHOOL: Engineering &amp; Technology</b>		<b>ACADEMIC SESSION: 2022 – 2023</b>		<b>FOR STUDENTS' BATCH: 2022-26 B. Tech. SEM-II</b>	
<b>1</b>	<b>Course code</b>	<b>BSC 105</b>			
<b>2</b>	<b>Course Title</b>	<b>Engineering Mathematics-II</b>			
<b>3</b>	<b>Credits</b>	<b>04</b>			
<b>4</b>	<b>Learning Hours</b>	<b>Lectures</b>		<b>03</b>	
		<b>Assessment OR Tutorial</b>		<b>01</b>	
		<b>Practical</b>			
		<b>Total hours</b>		<b>04</b>	
<b>5</b>	<b>Course Objective</b>	<p>After Studying this lesson, you should be able to:</p> <ol style="list-style-type: none"> <li>(1) To make the students understand the basic concepts, the history and tools of Co-ordinate Geometry of three dimension.</li> <li>(2) Explain importance of vector calculus and its applications.</li> <li>(3) Formulate and identify the characteristics of Fourier series Identify the various types of functions.</li> <li>(4) Understand the basics of second order linear differential equation.</li> </ol>			
<b>6</b>	<b>Course Outcomes</b>	<ol style="list-style-type: none"> <li>1. Conceptual knowledge of how three-dimension geometry and importance of sphere in engineering mathematics.</li> <li>2. To apply the concept of Rank of matrices and Caley-Hamilton Theorem on problems of linear equations. To implement the concept of eigen values and vectors in engineering problems</li> <li>3. The student has knowledge of central concepts in multivariable analysis, including space curves; directional derivative; gradient; multiple integrals; line and surface integrals; vector fields; divergence, curl and flux; the theorems of Green and Stokes, the divergence theorem and Fourier Series.</li> <li>4. Students learn to solve second order linear differential equations and partial differential equations of first order.</li> </ol>			
<b>7</b>					
	<b>Unit</b>	<b>Section</b>	<b>Introduction</b>	<b>Reference Number</b>	<b>Teaching Methods</b>
	<b>Unit-1</b>	(a)	<b>Coordinate Geometry of Three Dimensions:</b> Equation of a sphere, Intersection of a sphere and a plane, tangent plane, Intersection of two spheres, orthogonality of two spheres, right circular cone, right circular cylinder.	TB2 242-309	White Board & PPT

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<b>Unit - II</b>	(a)	<b>Matrices:</b> Rank of a matrix, Rank of matrix by reducing to normal forms,	TB3 111-131	White Board & PPT
	(b)	Consistency of systems of linear simultaneous equations and its solution,	TB3 155-183	White Board & PPT
	(c)	Eigen values and Eigen vectors, Cayley Hamilton theorem (without proof), Diagonalization of matrix.	TB3 234-270	White Board & PPT
<b>Unit-III</b>	(a)	<b>Vector Calculus:</b> Scalar and vector field, differentiation & integration of vector functions, Gradient, Divergence, Curl and Differential Operator, Line, Surface and volume Integrals.	TB1 372-430	White Board & PPT
<b>Unit-IV</b>	(a)	<b>Application of Vector Calculus:</b> Green's Theorem in a Plane, Gauss's and Stoke's Theorem (without proof) and their applications. <b>Fourier Series:</b> Expansion of simple functions in Fourier Series, half range Fourier sine and cosine series, change of interval. Harmonic Analysis.	TB1 430-466	White Board & PPT
<b>Unit-V</b>	(a)	<b>Differential Equations:</b> Series Solutions of Second Order Linear Differential Equations with Variable Coefficients (Complementary Functions only),	TB1 174-220	White Board & PPT
	(b)	Partial Differential Equations of First Order: Lagrange's Form, Standard Forms, Charpit's Method.	TB1 671-734	White Board & PPT
<b>8</b>	<b>Course Evaluation</b>			
<b>8.1</b>	<b>CA: 40%</b>			
<b>8.1.1</b>	<b>Attendance</b>	5%		
<b>8.1.2</b>	<b>Assignment &amp; presentation</b>	20%		
<b>8.1.3</b>	<b>Class test</b>	15%		
<b>8.1.4</b>	<b>Any other</b>	--		
<b>8.2</b>	<b>MTE</b>	20%		
<b>8.3</b>	<b>End-term examination: 40%</b>			
<b>9</b>	<b>Text Books &amp; References</b>			
<b>9.1</b>	<b>Text book</b>	<b>TB1:</b> H.K. Dass (1999). <i>Advanced Engineering Mathematics</i> , S Chand. <b>TB2:</b> A. R. Vasishth & D. C. Agrawal. <i>Analytical Geometry</i> . Krishna's. <b>TB3:</b> Advanced Engineering Mathematics, Erwin Kreyszig, Wiley 9th Edition. <b>TB4:</b> Calculus and Analytical Geometry, Thomas and Finney, Narosa Publishing		

		House N. Delhi. <b>TB5:</b> A Text Book of Differential Equations, M.Ray and Chaturvedi, Students Friends & Co. Publisher, Agra. <b>TB6:</b> Higher Engineering Mathematics, B.V.Ramana, Tata Mcgraw Hill.
9.2	References	<p><b>RB1:</b><a href="https://www.google.com/url?sa=t&amp;rct=j&amp;q=&amp;esrc=s&amp;source=web&amp;cd=&amp;cad=rja&amp;uact=8&amp;ved=2ahUKEwiqo4Srnrv3AhWP6mEKHQYgD0cQFnoECBMQAAQ&amp;url=http%3A%2F%2Fwww.numbertheory.org%2Fbook%2Fcha8.pdf&amp;usg=AOvVaw39xJwVU3SD7ZCEID-VQDAM">https://www.google.com/url?sa=t&amp;rct=j&amp;q=&amp;esrc=s&amp;source=web&amp;cd=&amp;cad=rja&amp;uact=8&amp;ved=2ahUKEwiqo4Srnrv3AhWP6mEKHQYgD0cQFnoECBMQAAQ&amp;url=http%3A%2F%2Fwww.numbertheory.org%2Fbook%2Fcha8.pdf&amp;usg=AOvVaw39xJwVU3SD7ZCEID-VQDAM</a></p> <p><b>RB2:</b><a href="https://www.google.com/url?sa=t&amp;rct=j&amp;q=&amp;esrc=s&amp;source=web&amp;cd=&amp;cad=rja&amp;uact=8&amp;ved=2ahUKEwi19o7Gnrv3AhUHZ94KHRZZDhQQFnoECAMQAAQ&amp;url=https%3A%2F%2Fwww.vmi.edu%2Fmedia%2Fcontent-assets%2Fdocuments%2Facademics%2Fappliedmath%2FFundamentals-of-Matrix-Algebra-3rd-Edition.pdf&amp;usg=AOvVaw1rz_afO9v4wCcZ_9fd-WkG">https://www.google.com/url?sa=t&amp;rct=j&amp;q=&amp;esrc=s&amp;source=web&amp;cd=&amp;cad=rja&amp;uact=8&amp;ved=2ahUKEwi19o7Gnrv3AhUHZ94KHRZZDhQQFnoECAMQAAQ&amp;url=https%3A%2F%2Fwww.vmi.edu%2Fmedia%2Fcontent-assets%2Fdocuments%2Facademics%2Fappliedmath%2FFundamentals-of-Matrix-Algebra-3rd-Edition.pdf&amp;usg=AOvVaw1rz_afO9v4wCcZ_9fd-WkG</a></p> <p><b>RB3:</b><a href="https://www.google.com/url?sa=t&amp;rct=j&amp;q=&amp;esrc=s&amp;source=web&amp;cd=&amp;cad=rja&amp;uact=8&amp;ved=2ahUKEwi19o7Gnrv3AhUHZ94KHRZZDhQQFnoECCsQAQ&amp;url=https%3A%2F%2Fwww.maths.ed.ac.uk%2F~v1ranick%2Fpapers%2Fgantmacher1.pdf&amp;usg=AOvVaw0e9OaZTvCra6imUznCPQlf">https://www.google.com/url?sa=t&amp;rct=j&amp;q=&amp;esrc=s&amp;source=web&amp;cd=&amp;cad=rja&amp;uact=8&amp;ved=2ahUKEwi19o7Gnrv3AhUHZ94KHRZZDhQQFnoECCsQAQ&amp;url=https%3A%2F%2Fwww.maths.ed.ac.uk%2F~v1ranick%2Fpapers%2Fgantmacher1.pdf&amp;usg=AOvVaw0e9OaZTvCra6imUznCPQlf</a></p> <p><b>RB4:</b><a href="https://www.google.com/url?sa=t&amp;rct=j&amp;q=&amp;esrc=s&amp;source=web&amp;cd=&amp;cad=rja&amp;uact=8&amp;ved=2ahUKEwjxkoLdnrv3AhVVdXAKHR2KAOYQFnoECAYQAQ&amp;url=https%3A%2F%2Fwww.robots.ox.ac.uk%2F~sjrob%2Fteaching%2FVector%2Fcourse.pdf&amp;usg=AOvVaw37VCFwUexnHTnSB-GUk4Cv">https://www.google.com/url?sa=t&amp;rct=j&amp;q=&amp;esrc=s&amp;source=web&amp;cd=&amp;cad=rja&amp;uact=8&amp;ved=2ahUKEwjxkoLdnrv3AhVVdXAKHR2KAOYQFnoECAYQAQ&amp;url=https%3A%2F%2Fwww.robots.ox.ac.uk%2F~sjrob%2Fteaching%2FVector%2Fcourse.pdf&amp;usg=AOvVaw37VCFwUexnHTnSB-GUk4Cv</a></p> <p><b>RB5:</b><a href="https://www.google.com/url?sa=t&amp;rct=j&amp;q=&amp;esrc=s&amp;source=web&amp;cd=&amp;cad=rja&amp;uact=8&amp;ved=2ahUKEwjxkoLdnrv3AhVVdXAKHR2KAOYQFnoECDAQAQ&amp;url=https%3A%2F%2Fwww.math.hkust.edu.hk%2F~machas%2Fvector-calculus-for-engineers.pdf&amp;usg=AOvVaw1Pah4khXOUAhJbxuz09kK6">https://www.google.com/url?sa=t&amp;rct=j&amp;q=&amp;esrc=s&amp;source=web&amp;cd=&amp;cad=rja&amp;uact=8&amp;ved=2ahUKEwjxkoLdnrv3AhVVdXAKHR2KAOYQFnoECDAQAQ&amp;url=https%3A%2F%2Fwww.math.hkust.edu.hk%2F~machas%2Fvector-calculus-for-engineers.pdf&amp;usg=AOvVaw1Pah4khXOUAhJbxuz09kK6</a></p> <p><b>RB6:</b><a href="https://www.google.com/url?sa=t&amp;rct=j&amp;q=&amp;esrc=s&amp;source=web&amp;cd=&amp;cad=rja&amp;uact=8&amp;ved=2ahUKEwjv3sX3nrv3AhWVEYgKHfnpBekQFnoECAIQAQ&amp;url=http%3A%2F%2Fmdudde.net%2Fpdf%2Fstudy_material_DDE%2FM.Sc.MAtematics%2FDIFFERENTIAL%2520EQUATIONS.pdf&amp;usg=AOvVaw2tNjEhWUEDFGGWzKJ72LEI">https://www.google.com/url?sa=t&amp;rct=j&amp;q=&amp;esrc=s&amp;source=web&amp;cd=&amp;cad=rja&amp;uact=8&amp;ved=2ahUKEwjv3sX3nrv3AhWVEYgKHfnpBekQFnoECAIQAQ&amp;url=http%3A%2F%2Fmdudde.net%2Fpdf%2Fstudy_material_DDE%2FM.Sc.MAtematics%2FDIFFERENTIAL%2520EQUATIONS.pdf&amp;usg=AOvVaw2tNjEhWUEDFGGWzKJ72LEI</a></p>
9.3	Video References	<p>[1] <a href="https://www.youtube.com/watch?v=VAiNh-5i6yA&amp;list=PLCvpYrhOPdiWKKx7cUFjwO9_mH6wxnKiv">https://www.youtube.com/watch?v=VAiNh-5i6yA&amp;list=PLCvpYrhOPdiWKKx7cUFjwO9_mH6wxnKiv</a></p> <p>[2] <a href="https://www.youtube.com/watch?v=KaLA1cWhQIA&amp;list=PLLy_2iUCG87BLK18eISe4fHKdE2_j2B_T">https://www.youtube.com/watch?v=KaLA1cWhQIA&amp;list=PLLy_2iUCG87BLK18eISe4fHKdE2_j2B_T</a></p> <p>[3] <a href="https://www.youtube.com/watch?v=aMlu6SakJW0&amp;list=PLbRMhDVUMngdcugfQ136Rnf50ki_8grvh">https://www.youtube.com/watch?v=aMlu6SakJW0&amp;list=PLbRMhDVUMngdcugfQ136Rnf50ki_8grvh</a></p> <p>[4] <a href="https://www.youtube.com/watch?v=ksS_yOK1vtk&amp;list=PLbRMhDVUMngfIrZCNOyPZwHUU1pP66vQW">https://www.youtube.com/watch?v=ksS_yOK1vtk&amp;list=PLbRMhDVUMngfIrZCNOyPZwHUU1pP66vQW</a></p> <p>[5] <a href="https://www.youtube.com/watch?v=XzaeYnZdK5o&amp;list=PLtKWB-wrvn4nA2h8TFxzWL2zy8O9th_fy">https://www.youtube.com/watch?v=XzaeYnZdK5o&amp;list=PLtKWB-wrvn4nA2h8TFxzWL2zy8O9th_fy</a></p> <p>[6] <a href="https://www.youtube.com/watch?v=NBcGLLU90fM&amp;list=PLbMVogVj5nJSGIf9sluucwoby_r_zz6g1D">https://www.youtube.com/watch?v=NBcGLLU90fM&amp;list=PLbMVogVj5nJSGIf9sluucwoby_r_zz6g1D</a></p> <p>[7] <a href="https://www.youtube.com/watch?v=U51IQtlzvA0&amp;list=PLgMDNELGJ1CZpu0rJvVh-bUHGFINS7xIk">https://www.youtube.com/watch?v=U51IQtlzvA0&amp;list=PLgMDNELGJ1CZpu0rJvVh-bUHGFINS7xIk</a></p>

**CO-PO Mapping**

Course Outcome	Program Outcome												PSO			
	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4
CO 1	1	3	3	3	1	2	1	1	2	2	1	2	3	1	2	2
CO 2	2	3	3	3	2	2	2	1	2	1	2	2	3	2	2	2
CO 3	2	3	3	3	2	2	1	1	2	1	2	3	3	2	2	2
CO 4	3	3	3	3	3	2	2	2	2	2	1	2	3	2	2	3

**Question Bank**

1) Find characteristic equation, eigen values and eigen vector of matrix B, where  $B = \begin{bmatrix} 5 & 0 & 0 \\ 2 & 0 & 0 \\ 1 & 3 & 5 \end{bmatrix}$ .

2) Find the equation to the sphere through the circle  $x^2 + y^2 + z^2 = 9$ ,  $2x + 3y + 4z = 5$  and the point (1, 2, 3).

3) Find the equation to the sphere through the circle  $x^2 + y^2 + z^2 = 1$ ,  $2x + 4y + 5z - 6 = 0$  and touching the plane  $z = 0$ .

4) Find rank of the following matrices.

I.  $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 2 & 2 \end{bmatrix}$

II.  $\begin{bmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{bmatrix}$

5) Reduce the matrix A to its normal form and find the rank of A.

$$A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

6) Find characteristic equation, eigen values and eigen vector of matrix B, where  $B = \begin{bmatrix} 5 & 0 & 0 \\ 2 & 0 & 0 \\ 1 & 3 & 5 \end{bmatrix}$ .

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- 7) Find the equation to the sphere through the circle  $x^2 + y^2 + z^2 = 9$ ,  $2x + 3y + 4z = 5$  and the point  $(1, 2, 3)$ .
- 8) Find the equation to the sphere through the circle  $x^2 + y^2 + z^2 = 1$ ,  $2x + 4y + 5z - 6 = 0$  and touching the plane  $z = 0$ .
- 9) Find equation of sphere for which the circle  $x^2 + y^2 + z^2 + 7y - 2z + 2 = 0$ ,  $2x + 3y + 4z = 8$  is great circle.
- 10) Find equation of sphere for which the circle  $x^2 + y^2 + z^2 + 7y - 2z + 2 = 0$ ,  $2x + 3y + 4z = 8$  is great circle.
- 11) If A and B are two square matrices. Then  $(AB)^t =$  \_\_\_\_\_

$B^t A^t \quad A^t B^t \quad AB \quad -B^t A^t$

- 12) For a matrix B,  $(B^t)^t$  is equal to \_\_\_\_\_

$-B \quad B \quad B^t \quad -B^t$

- 13) If A is a singular matrix then  $|A| =$  \_\_\_\_\_

14) Find  $\begin{vmatrix} 3 & 0 & 5 \\ 0 & 1 & 3 \\ 0 & 0 & 7 \end{vmatrix} =$  \_\_\_\_\_

- 15) A square matrix  $A = [a_{ij}]_{n \times n}$  is said to be symmetric if

- a)  $a_{ij} = a_{ji}$  for  $i, j \in \{1, 2, \dots, n\}$
- b)  $a_{ij} = -a_{ji}$  for  $i, j \in \{1, 2, \dots, n\}$
- c)  $a_{ij} = 0$  for  $i = j, i \in \{1, 2, \dots, n\}$
- d) None of the above

16) If  $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$  then  $9A$  is \_\_\_\_\_

17) A square matrix  $A = [a_{ij}]_{n \times n}$  is said to be skew-symmetric if

- a)  $a_{ij} = a_{ji}$  for  $i, j \in \{1, 2, \dots, n\}$
- b)  $a_{ij} = -a_{ji}$  for  $i, j \in \{1, 2, \dots, n\}$
- c)  $a_{ij} \neq 0$  for  $i = j, i \in \{1, 2, \dots, n\}$
- d) None of the above

18) Which of the matrix is a scalar matrix?

- a)  $\begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$
- b)  $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$
- c)  $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
- d)  $\begin{bmatrix} 0 & 1 \\ 4 & 0 \end{bmatrix}$

19) If  $A$  and  $B$  are square matrices then  $(AB)' =$  \_\_\_\_\_

- a)  $B'A'$
- b)  $A'B'$
- c)  $AB$
- d)  $BA$

20) Q.11 If  $\begin{bmatrix} 1-x & 2 \\ 18 & 6 \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 18 & 6 \end{bmatrix}$  then  $x =$

21) If  $\begin{vmatrix} x & 5 \\ 3 & 3 \end{vmatrix} = 0$ , then value of  $x$  is \_\_\_\_\_

22) If  $A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$  and  $B = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$  then  $AB =$  \_\_\_\_\_

$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$        $\begin{bmatrix} 3 & 1 \\ 1 & 0 \end{bmatrix}$        $\begin{bmatrix} 3 & 1 \\ -1 & -1 \end{bmatrix}$        $\begin{bmatrix} 3 & 0 \\ -1 & -1 \end{bmatrix}$

23) If  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & b-4 & 7 \\ a-7 & 6 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 7 \\ 4 & 6 & -1 \end{bmatrix}$  then the value of  $a$  and  $b$  respectively are  $a=11, b=5$

24) If  $A = \begin{bmatrix} 1 & 2 & -1 \\ 4 & 3 & 2 \\ 1 & 2 & -1 \end{bmatrix}$  then

- $|A|=0$        $|A| \neq 0$        $A^{-1}$  exists      none of the above

25) If  $A = \begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix}$  then  $|2A| =$

26) For matrix  $A$ ,  $A^3 = I$ ,  $A^{-1}$  is equal to \_\_\_\_\_

$A^2 \quad A^{-2} \quad A \quad A^{-1}$

27) If  $A$  is an invertible matrix then  $|A|$  is always \_\_\_\_\_

$\neq 0 \quad = 0 \quad = 3 \quad \text{none of these}$

28) Consider the statements

(a)  $2x + 3y = 0$

$4x + 5y = 0$  is a homogeneous system of equations

$3x + 10y = 9$

(b)  $3x + y = 0$

$4x + 10y = 0$  is a non-homogeneous system of equations

$3x + 7y = 0$

29) The value of  $\left(\frac{\partial z}{\partial x}\right)_{(1,-1,2)} =$  \_\_\_\_\_, if  $x^2 + y^2 + z^2 = a^2$ .

(a) 2      (b) 1      (c) 4      (d)  $-\frac{1}{2}$

30) If  $z = f(x, y)$  is homogeneous function of  $x, y$  of degree 6, then

$x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} =$  \_\_\_\_\_

(a)  $6z$       (b)  $30z$       (c)  $30$       (d)  $6$

31)  $\lim_{h \rightarrow 0} \frac{f(x+h,y) - f(x,y)}{h}$ , exists if:

(a)  $x$  at  $(x, y)$       (b)  $x$  at  $(a, b)$       (c)  $y$  at  $(x, y)$       (d)  $y$  at  $(a, b)$

32) If  $(x, y) = x^3 + y^3 - 2x^2y^2$ , then  $(f_{xx})_{x=y=1} =$

(a) 1      (b) -1      (c) 0      (d) None of these

33) Find the total differential coefficient of  $x^2y$  with respect to  $x$  when  $x, y$  connected by

$x^2 + xy + y^2$ .

34) Find  $\frac{d^2y}{dx^2}$  for  $x^5 + y^5 = 5a^3x^2$ .

35) Use Taylor's theorem to expand  $f(x, y) = x^2 + xy + y^2$  in the powers of  $(x - 1)$  and  $(x - 2)$ .

Use the method of separation of variables to find the general solution to the following

differential equation.  $u' = \frac{1}{5 - 2u}$

36) Using the existence and uniqueness theorem for second order linear ordinary differential equations, find the largest interval in which the solution to the initial value is certain to exist.

$$t(t-4)y'' - ty' + 3t^2y = 0, y(1) = 1, y'(1) = 3.$$

37) Find the solution to the given initial value problem.

$$4y'' - 12y' + 9y = 0, y(0) = 0, y'(0) = 1$$

38) Evaluate  $\iint_R y^2 dydx$ ; where  $R$  is the region bounded by the curves  $y^2 = x$  and  $y = x^3$ .

39) Evaluate  $\iint_R y^2 dx dy$ ; where  $R$  is the region bounded by the curves  $y^2 = x$  and  $y = x^3$

40) Integrate  $f(x, y) = x/y$  over the region in the first quadrant bounded by the lines  $y = x$ ,  $y = 2x$ ,  $x = 1$ ,  $x = 2$ .

41) Integrate  $f(x, y) = x^2 + y^2$  over the triangular region with vertices  $(0, 0)$ ,  $(1, 0)$  and  $(0, 1)$ .

42) Evaluate  $\iint_R (x + y) dydx$ ; where  $R$  is the region bounded by the curves  $x = 2$ ,  $y = x$ ,  $y = x + 2$  and  $x = 0$ .

43) Evaluate  $\iint_R (x + y) dx dy$ ; where  $R$  is the region bounded by the curves  $x = 2$ ,  $y = x$ ,  $y = x + 2$  and  $x = 0$ .

44) Evaluate  $\iint_R \sqrt{xy - y^2} dA$ ; where  $R$  is the region bounded by the triangle with vertices  $(0, 0)$ ,  $(10, 1)$  and  $(1, 1)$ .

45) Evaluate  $\iint_R r^3 dr d\theta$ ; where  $R$  is the region bounded by the curves  $r = 2 \sin \theta$  and  $r = 4 \sin \theta$ .

46) Evaluate  $\iint_R r^3 dr d\theta$ ; where  $R$  is the region bounded by the curves  $r = 2 \cos \theta$  and  $r = 4 \cos \theta$ .

47) If  $A = \begin{bmatrix} 6 & -5 & 2 \\ 0 & 5 & -2 \end{bmatrix}$  and  $B = \begin{bmatrix} 3 & 0 \\ -4 & -2 \\ 1 & -4 \end{bmatrix}$ , then find  $AB$  and  $BA$ . Show that  $AB \neq BA$ .

48) Find  $X$  and  $Y$ , if  $X + Y = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix}$  and  $X - Y = \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$

49) Find rank of the following matrices



a)  $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 2 & 2 \end{bmatrix}$

b)  $\begin{bmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{bmatrix}$

50) Find characteristic equation, eigen values and eigen vector of matrix B, where  $B = \begin{bmatrix} 6 & 0 & 8 \\ 0 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

51) Reduce the matrix A to its normal form and find the rank of matrix  $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ .