RAFFLES UNIVERSITY

TEACHING PLAN: GRAPH THEORY & COMBINATORICS

SCHOOL: (SOEAT) SCHOOL OF ENGINNERING & TECHNOLOGY	ACADEMIC SESSION: 2022 – 2023	FOR STUDENTS' BATCH: 2021-25 B.Tech Sem-IV

1	**Course No.**	**PCC- CSE 209**
2	**Course Title**	**Graph Theory & Combinatorics**
3	**Credits**	**5**
4	**Learning Hours**	**Contact Hours**: 4 **Practical Teaching**: 0 **Projects, Tutorials and Assessments**: 1 **Total Hours**: 5
5	**Course Objective**	By learning Graph Theory & Combinatorics students able to do each of the following: 1. Investigate functions as relations and their properties. 2. To provide a first approach to the subject of algebra, which is one of the basic pillars of modern mathematics and to study of certain structures called groups, rings, fields and some related structures. 3. To develop the under-standing of Geometric duals in Planar Graphs. 4. To understand the concept of matrices in graphs like Incidence matrix, Adjacency matrix, Cycle matrix etc 5. Explain some of the concepts of number theory, a primary area of mathematics, using examples.
6	**Course Outcomes**	CO1 – In this course, the basic pillars of modern mathematics will be introduced and analysed. These structures include groups, rings, fields, any mapping between them and their substructures. CO2 – To understand and apply the fundamental concepts in graph theory and to apply graph theory-based tools in solving practical problems. CO3 - Apply mathematical ideas and concepts within the context of number theory and communicate number-theoretic techniques to a mathematical audience. CO4 - This course will give students the combinatorial tools to model and analyse practical problems in various areas.
7	**Outline syllabus:**	

	Paper Code	Unit	Introduction	Page Numbers	Teaching Aids
	PCC- CSE 209	Unit-1: Set Theory	Operations on sets, Laws of algebra of sets, Representation of sets on computer in terms of 0‟s & 1‟s. Partition & covering of a set, ordered pair, Product set, Relation– Different types of relations, Graph of relation, Matrix of relation, Transitive closure of relation, Properties of relations, Compatible relation. Functions, Partial ordering & partially ordered set, Hasse diagram of Poset, totally ordered set, Peano axioms & Mathematical Induction.	TB5: Ch 2 – Ch 10	White Board, PPT Slides
		Unit II: Group	Algebra or Algebraic systems like semi group, monoid, and examples. Homomorphism, Isomorphism of semi group, monoid. Groups, properties of algebraic groups. Permutations groups, Subgroups, Co sets, Lagranges	TB6: 27-230	White Board, PPT Slides

			theorem, properties of cyclic groups, generator of group, kernel of Homorphism, quotient group, fundamental theorems & Homomorphism of groups, Residue classes & Fermats theorem.		
		Unit III: Rings	Rings, types of rings, Fields, subring, Integral domain. Simple properties of rings. Lattice as Poset & as algebraic system, Types of lattices, Hasse diagrams, Sublattice, direct product of Lattices, Lattice Homomorphism, complement of elements of lattices, Various lattices, composition tables, Lattice Bn . Boolean algebra; Boolean Expressions, Equivalence of Boolean ression by tables, Simplification of circuit & equivalent circuit by truth tables.	TB6: 237-322	White Board, PPT Slides
		Unit IV: Graph Theory	Graphs and its types, subgraph, Quotient graph, Euler path, Complete path, in degree out degree, reach ability, cycle, matrix representation of graph. Transitive closure of graph, Adjacency matrix, Trees, Venn diagram, Representation of trees, binary trees, spanning trees, Prims algorithm.	TB7: 1-96	White Board, PPT Slides
		Unit V: Combinatorics	Definition of generating functions and examples, proof of simple combinatorial identities, Probab. G.F. , examples. Recursive relations: definitions= =∑p(t) p t , E(x) p (t) n , n & examples, explicitly formula for sequence, back tracking to find explicit formula of sequence, solving recurrence relations. Counting Thjeorem and application, Equivalent sets, cardinal numbers, denumerable sets. Multiplication principle of counting. Permutation & Combination with examples. The pigeon hole principle & extended pigeon hole principle and application of pigeon hole principle in solving simple problems.	TB8: 43-149	White Board, PPT Slides
		Unit VI: Number Theory	Examples of continued fractions. The study of continued fractions. alpha has Infinite continued fraction if alpha is irrational. Alpha has periodic continued fractions if alpha is quadratic irrational. Application to approximation of irrationals by rationals. Hurwitz"s theorem.	TB 9: 75-129	White Board, PPT Slides

8	**Course Evaluation**	
8.1	**CA: 20%**	
8.11	**Attendance**	10%
8.12	**Homework**	2 Assignments, 10%
8.13	**Quizzes**	-
8.14	**Projects**	-
8.15	**Presentation**	-
8.13	**Any other**	-
8.2	**MTE**	20%
8.3	**End-term examination: 60%**	
9	**Text Books & References**	
9.1	**Text book**	TB1. D. B. West, Introduction to graph theory; Prentice Hall TB2. S. Pirzada, An introduction to graph theory; Universities Press, Orient. Blackswan, 2013. TB3. “Topics in Algebra” by I.N. Herstein, Wiley. TB4: Real Analysis by. H. L. Royden TB5: A book of set theory by Charles C. Pinter, DOVER PUBLICATIONS, INC. TB6: Contemporary abstract algebra by Joseph A. Gallian. TB7: Introduction to Graph Theory by Robin J. Wilson, Longman. TB8: Discrete Mathematics by Seymour Lipschutz & Marc Lars Lipson, Schaum’s outlies. TB9: Number Theory by Z. I. Borevich and R. I. Safarevich

*****Best Of Luck*****

9.2	**References**	1. Borevich, Z. I., & Shafarevich, I. R. (1986). Number theory. Academic press. 2. Hua, L. K. (2012). Introduction to number theory. Springer Science & Business Media. 3. Scott, W. R. (2012). *Group theory*. Courier Corporation. 4. Rose, J. S. (1994). *A course on group theory*. Courier Corporation. 5. Biggs, N., Lloyd, E. K., & Wilson, R. J. (1986). Graph Theory, 1736-1936. Oxford University Press. 6. West, D. B. (2001). *Introduction to graph theory* (Vol. 2). Upper Saddle River: Prentice hall.
9.3	**Video References**	1) https://www.youtube.com/watch?v=flQsH3UYRsA&list=PLyqSpQzTE6M99lRJsSpAyKg49IjPO-vD6 2) https://www.youtube.com/watch?v=VdLhQs_y_E8&list=PLelIK3uylPMGzHBuR3hLMHrYfMqWWsmx5 3) https://www.youtube.com/watch?v=u7cBLb0b7pk&list=PLEAYkSg4uSQ1zCXoJK7ZtWbz_WGmlLL1- 4) https://www.youtube.com/watch?v=E40r8DWgG40&list=PLEAYkSg4uSQ2fXcfrTGZdPuTmv98bnFY5 5) https://www.youtube.com/watch?v=yKRbG9Y5pYY&list=PLEAYkSg4uSQ3AaON5oCbS6ecwKsoopBN3

CO-PO Mapping

Course Outcome	Program Outcome												PSO			
	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4
CO 1	1	3	3	3	1	2	1	1	2	2	1	2	3	1	2	2
CO 2	2	3	3	3	2	2	2	1	2	1	2	2	3	2	2	2
CO 3	2	3	3	3	2	2	1	1	2	1	2	3	3	2	2	2
CO 4	2	3	3	3	2	2	2	1	2	2	1	2	3	2	2	2

QUESTION BANK

1) Prove that Z_6 is cyclic group and find all its subsets.

2) G is group such that $(a.b)^2 = a^2.b^2$ for all $a, b \in G$, if and only if G is abelian.

3) Find the number of mathematics students at a collage taking at least one of the languages French, German and Russian given the following data

   - 65 study French
   - 45 study German
   - 42 study Russian
   - 20 study French and German
   - 25 study French and Russian
   - 15 study German and Russian
   - 8 study all the three languages

4) Consider the sets $A_1 = \{2,3,4,...\}$, $A_2 = \{3,4,5,...\}$, $A_3 = \{4,5,6,...\}, \ldots, A_n = \{n+1, n+2, n+3, ...\}$.

   Then find the following

*****Best Of Luck*****

I. $\bigcup(A_n : n \in N)$

II. $\bigcap_{i=1}^{20}(A_i : i \in N)$

III. $\bigcap_{i=1}^{10} A_i$, where i is odd

IV. $\bigcap(A_n : n \in N)$

V. $\bigcup_{i=1}^{20}(A_i : i \in N)$

5) Find power sets of set $A = \{1, 2, 3, 4\}$.

6) Prove associative and DeMorgan's laws using truth table.

7) Consider the Venn diagram of three arbitrary sets A, B and C and shade the region which describe the following: $(A \cap B^C) \cap C$ II. $(A \setminus B^C) \setminus C$

8) Let G be a finite group of order $2n$ for some integer n. Consider themap $\varphi : G \to G$ given by $\varphi(a) = a^2$ then φ is
   a. injective
   b. surjective
   c. injective but not surjective
   d. not surjective

9) Suppose that $G = \{e, x, x^2, y, yx, yx^2\}$ is a non-abelian group with $|x| = 3$ and $|y| = 2$, then
   a. $xy = yx$ (b) $xy = yx^2$ (c) $xy = x^2y$ (d) $yx = yx^2$

10) The number of cyclic subgroup of order 10 in $Z_{100} \otimes Z_{25}$. (a) 24 (b) 20 (c) 6 (d) 2

11) Which of the following is not true?
   a. There is group G with a proper subgroup H such that G and H are isomorphic.
   b. there is a non-cyclic group G such that every non identity elementof G has order 5.
   c. Q^* under multiplication is not isomorphic to R^* under multiplica-tion.
   d. Q under addition is isomorphic to R under addition.

12) Number of homomorphism from A_n to C^* is

   (a) 4 (b) 3 (c) 1 (d) 2

13) Order of $Aut(D_6)$ is

(a) 10 (b) 36 (c) 12 (d) 2

14) Consider a abelian group G of order 24. The number of subgroups oforder 8 in G are

(a) 1 (b) 6 (c) 2 (d) 3

15) Which of the followings is/are not necessarily true?

a. If center of a group G is equal to group G then the group G iscyclic.
b. Number of elements of order 8 in S_7 are 630.
c. $\mathbb{R}\backslash 2\pi\mathbb{Z}$ is isomorphic to the group of complex numbers havingmodulus 1 with multiplication.
d. Let G be a group, then there is a subgroup H of G such that

$$N(H)\backslash C(H) \text{ is isomorphic to } Aut(H)$$

16) Let G be a group with the generators a and b which satisfies $a^4 = b^2 = e$ and $aba = b$. Then $\frac{G}{Z(G)}$ is isomorphic to

a. the trivial group
b. $\mathbb{Z}_2 \times \mathbb{Z}_2$
c. $\mathbb{Z}_2$
d. $\mathbb{Z}_4$

17) Which of the following statement is true?

a. Let $f : \mathbb{Q} \to \mathbb{Q}$ be a non-trivial homomorphism then f is anisomorphism.
b. If group G is finite, then $Aut(G)$ is finite.
c. If group G is cyclic, then $Aut(G)$ is cyclic.
d. For some $n \in \mathbb{N}, n \geq 4, A_n$ is not simple.

18) Prove A group of order 289 is cyclic.

19) The permutation group S_{12} has an element of order

(a) 16 (b) 36 (c) 28 (d) 42

20) Which of the following is not true?

a. There exists no infinite group in which every element has a finite order.

b. If a group of order p^n, p-prime, contains exactly one subgroup of order $p, p^2, p^3 \cdots p^{n-1}$, then it is cyclic.

c. There are exactly two non abelian group of order 8.

d. If H is a subgroup of a group G such that $x^2 \in H$ for every $x \in G$ then $G \backslash H$ is abelian.

21) Consider the system of congruences $x \equiv 3 (\text{mod } 17)$ and $x \equiv 2 (\text{mod } 119)$then the given system has

a. a unique solution to the given system.

b. no solution to the given system.

c. infinitely many solutions to the given system.

d. a solution in the interval [240, 440].

22) The number of positive integers ≤ 100 which are divisible by at least oneof the prime $2, 5, 11$ is

(a) 64 (b) 60 (c) 80 (d) 63

23) Let G be a group generated by the elements x and y with the relations $x^5 = y^2 = 1$ and $y^{-1}xy = x^2$. The order of G is

(a) 3 (b) 2 (c) 4 (d) 5

24) Let G be a finite group and N be a normal subgroup of G of index p, aprime number. Then the number of all subgroups of G containing N is

(a) p (b) $2p$ (c) 2 (d) 1

25) Which of the followings is/are not necessarily true?

a. The group $GL_2(\mathbb{R})$ contains a cyclic subgroup of order 7.

b. In symmetric group $S_n$any two elements of same order are conjugate.

c. If every proper subgroup of a infinite group G is cyclic then G is a cyclicgroup.

d. There is a group G such that order of $G/Z(G)$ is a prime number.

26) Let $a_n$denote the number of those permutation σ on $\{1, 2, 3..., n\}$ such that σ is a product of at least three disjoint cycles. Then

(a) $a_5 = 46$ (b) $a_5 = 45$ (c) $a_4 = 6$ (d) $a_4 = 7$

27) Let G be a group of order 30 and H a normal subgroup of G. what arepossible order of G/H?

(a) 15 (b) 10 (c) 5 (d) 6

28) Which of the following group have trivial center?

a. A group of order p^4, p is prime.
b. A group of invertible $n \times n$ matrices of over R.
c. A group G such that $G/Z(G)$ is cyclic.
d. A permutation group S_n, $n > 7$

29) Consider the statements

(a) Every Monoid is a semigroup.

(b) Every semigroup is a monoid.

Which of the following statements is/are always true?

1. Only (a)
2. Only (b)
3. Both (a) and (b)
4. Neither (a) nor (b)

30) Which of the following is not compound statement

1. Today is a rainy day but I will go to college
2. 2+4=9
3 I have a blue pen or a red pen
4 Today is Monday but it is 2nd February

31) Let S= {a, b} and aRb. Which condition is satisfied for R to be symmetric?

1. a=-b, b=-a
2. a=-a, b=-b
3. a=b, b=a
4. a=b, b ≠a

32) For real numbers x and y, $xy \neq 0$ truthiness for $xy \neq 0$ is possible in case of

(a) $x = 1, y = 0$
(b) $x = 0, y = 1$
(c) $x = 1, y = -3$
(d) $x = 0, y = -2$

33) x is greater than y, z and R is ____-place predicate.

(a) 1
(b) 2
(c) 3
(d) 4

34) Let $S = \{2, -1\}$. Which of the following is a binary operation on S .

1 For $x, y \in S$, xy

2. For $x, y \in S$, $x + y$

3. For $x, y \in S$, $x - y$

4 None of the above

35) Which of the following is a partition of the set $A = \{3,4,5,6\}$

1 $A_1 = \{ \{3,4\}, \{5\}, \{6\}\}$

2 $A_2 = \left\{\{3,4,5\},\ \{5,6\}\right\}$

3 $A_3 = \{ \{3\},\ \{3,4,5\}\}$

4 $A_4 = \{ \{3,4\},\ \{3,4,5\}\}$

36) The number of equivalence classes of congruence module 11 are

1 6

2 11

3 9

4 5

37) Which of the following is a commutative binary operation on set of all irrational numbers.

a. Addition

b. Subtraction

c. Multiplication

Both option (c) and (b)

Only option (b) Ans: 2

Both option (a) and (c)

None

38) Which of the following is the multiplicative identity of set of rational numbers with respect to multiplication

1. 0

2. 2

3 -1

4 1

39) P: Mark is Rich

Q: Mark is Unhappy

“Mark is rich or he is rich and happy”

Which of the following is the symbolic form of above statement

1. $P \vee (P \wedge Q)$

2. $P \vee (\sim P \vee Q)$
3. $P \vee (P \wedge \sim Q)$
4. $P \vee (P \vee \sim Q)$

40) Which of the following is a contradiction:

1 . $P \vee P \equiv P$ 2 $P \vee (\sim P)$ 3 $\sim\sim P \equiv P$ 4. $P \wedge (\sim P)$

41) Negation of the statement "for all $y \in \mathbb{R}$, $y^3 > 0$ "

1. There exists $y \in \mathbb{R}$ such that $y^3 < 0$
2 There exists $y \in \mathbb{R}$ such that $y^3 > 0$
3 For all $y \in \mathbb{R}$, $y^3 < 0$
4 None of the above

42) For real numbers x and y, $xy < 0$ truthiness for $xy < 0$ is possible in case of
a) $x = 1, y = 0$
b) $x = 0, y = 1$
c) $x = 1, y = -3$
d) $x = -1, y = -2$

43) x is less than y and z is _____-place predicate.
a) 1 b) 2 c) 3 d) 0

44) Which of the following expression is for a bi-conditional statement?

1 If P then Q
2 Q if P
3 $P \wedge Q$
4 $P \rightleftarrows Q$

45) If $P \rightleftarrows Q$ is true then the truth values of P and Q are respectively:

1 F, F   2 T, F   3. F, T   4. none

46) Consider A={1,2,3} Which of the following is a symmetric relation on A

1 R= $\{(x, y)|\ x < y,\ x, y \in A\}$
2 S= $\{(x, y)|\ x \leq y,\ x, y \in A\}$
3 P= $\{(x, y)|\ x > y\ or\ x < y,\ x, y \in A\}$
4 R= $\{(x, y)|\ x \geq y,\ x, y \in A\}$