TEACHING PLAN: NUMERICAL METHODS & ANALYSIS

SCHOOL: Engineering & Technology	ACADEMIC SESSION: 2022 – 2023	FOR STUDENTS' BATCH: 2020-24 B. Tech. SEM-VI
1	**Course code**	**PEC-ME-303**
2	**Course Title**	**Numerical Methods & Analysis**
3	**Credits**	**04**
4	**Learning Hours**	**Lectures**: **03** **Assessment OR Tutorial**: **01** **Guided Study**: **Total hours**: **04**
5	**Course Objective**	After Studying this lesson, you should be able to: (1) Demonstrate understanding of common numerical methods and how they are used to obtain approximate solutions to otherwise intractable mathematical problems. (2) Apply numerical methods to obtain approximate solutions to mathematical problems. (3) Derive numerical methods for various mathematical operations and tasks, such as interpolation, differentiation, integration, the solution of linear and nonlinear equations, and the solution of differential equations.
6	**Course Outcomes**	1. To explore complex systems, physicists, engineers, financiers and mathematicians require computational methods since mathematical models are only rarely solvable algebraically 2. Analyse and evaluate the accuracy of common numerical methods. 3. The course will cover the classical fundamental topics in numerical methods such as, approximation, numerical integration, numerical linear algebra, solution of nonlinear algebraic systems and solution of ordinary differential equations.
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	Unit	**Section**	**Introduction**	**Reference Number**	**Teaching Methods**
	Unit-1	(a)	Basic concepts: round-off errors, floating point arithmetic, Convergence. Numerical solution of Nonlinear Equations a) Bisection method, fixed-point iteration, Newton's method, Error analysis for Iterative Methods, Computing roots of polynomials.	TB4: 1-43	White Board & PPT
	Unit - II	(a)	Interpolation and Polynomial Approximation: LaGrange Polynomial, Divided Differences, Hermite Interpolation	TB4: 73-125	White Board & PPT
	Unit-III	(a)	Numerical integration and differentiation: Trapezoidal rule, etc., Gaussian quadrature and Euler-Maclaurin formula. Applied Linear Algebra, Direct methods for solving linear systems, numerical factorizations, Eigenvalue	TB4: 207-254	White Board & PPT

			problems.		
	Unit-IV	(a)	IVP problems for ODE:Euler's, Taylor, Runge-Kutta, and multistep methods, Stability,Numerical linear algebra,Direct methods,Iterative methods	TB4: 302-341	White Board & PPT
	Unit-V	(a)	Approximation theory: Least square approximation,Approximating Eigenvalues,Power method, Householder's method , BVP for ODE ,Shooting method	TB4: 405-430	White Board & PPT

8	**Course Evaluation**	
8.1	**CA: 40%**	
8.1.1	**Attendance**	5%
8.1.2	**Assignment & presentation**	20%
8.1.3	**Class test**	15%
8.1.4	**Any other**	--
8.2	**MTE**	20%
8.3	**End-term examination: 40%**	
9	**Text Books & References**	
9.1	**Text book**	**TB1:** J. Stoer and R. Bulirsch, Introduction to Numerical Analysis, Springer-Verlag, ISBN 0-387- 90420-4 **TB2:** L.N. Trefethen and D. Bau, Numerical Linear Algebra, Society of Industrial and Applied Mathematics **TB3:** C.T. Kelley, Iterative methods for linear and nonlinear equations, Society of Industrial and Applied Mathematics **TB4:** Introductory Methods of Numerical Analysis by S. S. Sastry, PHI Learning.
9.2	**References**	**RB1:** Isaacson, E., & Keller, H. B. (2012). *Analysis of numerical methods*. Courier Corporation. **RB2:** Chapra, S. C., & Canale, R. P. (2011). *Numerical methods for engineers* (Vol. 1221). New York: Mcgraw-hill. **RB3:** Dahlquist, G., & Björck, Å. (2003). *Numerical methods*. Courier Corporation. **RB4:** Epperson, J. F. (2021). *An introduction to numerical methods and analysis*. John Wiley & Sons. **RB5:** Kahaner, D., Moler, C., & Nash, S. (1989). *Numerical methods and software*. Prentice-Hall, Inc..
9.3	**Video References**	1) https://www.youtube.com/watch?v=zT83sJ5IrEE&list=PLyqSpQzTE6M-QT7PvEBHV0iNMvZk9mocO 2) https://www.youtube.com/watch?v=pOtnzAXlXvI&list=PL3pGy4HtqwD0CWdFuygdF-gk0ORk5EFZg 3) https://www.youtube.com/watch?v=JPSi-WCOhk4&list=PLoFGL7wppr4tdWBUS-wj-J1AHIVz21fTB 4) https://www.youtube.com/watch?v=zjyR9e-N1D4&list=PL1A70C686CB3C95FC 5) https://www.youtube.com/watch?v=TWAN_T66Cps&list=PLq-Gm0yRYwTguDcfylj1ZicXxzdZCAr5S

CO-PO Mapping

Course Outcome	Program Outcome												PSO			
	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4
CO 1	1	3	3	3	1	2	1	1	2	2	1	2	3	1	2	2
CO 2	2	3	3	3	2	2	2	1	2	1	2	2	3	2	2	2
CO 3	2	3	3	3	2	2	1	1	2	1	2	3	3	2	2	2

Question Bank

1) Estimate the maximum error occurring in approximation of $\ln(2.7)$ using interpolating polynomial passing through $(2,\ 0.69315)$, $(2.5,\ 0.91629)$ and $(3,\ 1.09861)$?
2) Estimate the maximum error occurring in approximation of $\sin(\pi/6)$ using interpolating polynomial passing through (0, 0), ($\pi/4$, 0.70711) and ($\pi/2$, 1)?
3) Estimate the maximum error which may be occurred while evaluating $\int_0^1 e^x$ using trapezoidal rule considering 14 subintervals.
4) Fins the approximate step size so that the value of the integral $\int_0^1 e^x\,dx$, using Simpson's one third rule is correct up to 6 decimal places.
5) Using Euler's method to find the solution of the differential equation (IVP) $xy' = x - y$, $y(2) = 2$ at 2.3 correct to five decimal places.
6) Given the initial value problem defined by $\frac{dy}{dx} = y(1 + x^2)$, $y(0) = 1$, find the values of y for $x = 0.2$, 0.4, 0.6 and 0.8 and $x = 1.0$ using the Euler's method, correct to 4-decimal places.
7) Determine the value of y when $x = 0.1$ given that $y(0) = 1$ and $y' = x^2 + y$, correct up to 3-decimal places, considering $h = 0.05$.
8) Using Modified Euler's method to find the solution of the differential equation (IVP) $xy' = x - y$, $y(2) = 2$ at 2.3 correct to five decimal places.
9) Given the initial value problem defined by $\frac{dy}{dx} = y(1 + x^2)$, $y(0) = 1$, find the values of y for $x = 0.2$, 0.4, 0.6 and 0.8 and $x = 1.0$ using the Modified Euler's method, correct to 4-decimal places.
10) Given $\frac{dy}{dx} = y - x$ where $y(0) = 2$, find $y(0.4)$ considering $h = 0.1$ (step size 0.1), correct up to 3-decimal places.
11) Using R-K (2$^{\text{nd}}$ order) method, find the solution of the differential equation (IVP) $xy' = x - y$, $y(2) = 2$ at 2.3 correct to five decimal places.
12) Given the initial value problem defined by $\frac{dy}{dx} = y(1 + x^2)$, $y(0) = 1$, find the values of y for $x = 0.2$, 0.4, 0.6 and 0.8 and $x = 1.0$ using the Euler's method, correct to 4-decimal places.
13) Fill in the blanks with the most appropriate answer, from the given alternatives :
    i. For finding $y(19)$ using Stirling's interpolation formula, with the given data points $(5,15)$, $(10,7)$, $(15,1)$, $(20,-5)$ and $(25,-12)$, the central abscissa x_0=________________. (20, 15, 19)

ii. (Approximation of $\frac{1}{30}$, correct up to 3 significant digits is ____________. (0.033, 0.03, 0.0333)

iii. A root of the equation $10 \sin x + x = 0$ is in the interval _____________. $((5°, 6°)$, $(5^c, 6^c)$, $(4°, 5°))$

iv. Let 2.57123 is an exact solution and 2.57157 an approximate solution of equation $f(x) = 0$ then the relative error is ____________. $(0.00034, 0.00013, 0.01322)$

14) Giving reasons, state in the box provided whether the following statements are TRUE or FALSE:
   i. Let a root of equation $f(x) = 0$ is in the interval $(-1,0)$ then number of iteration required, for finding approximate root correct up to three decimal places, are 10.
   ii. Let Δ, ∇ and E are forward difference, backward difference and shift operator respectively then $\Delta + \nabla + E^{-1} \equiv E$.

15) Define the terms, "algebraic equation" and "transcendental equation".

16) Show that the coefficient of x^5 in the interpolating polynomial of the date points $(0.2,1)$, $(0.4,2)$, $(0.6,3)$, $(0.8,4)$, $(1,5)$ and $(1.2,6)$ is zero.

17) Show that $\Delta^n \sin(a + bx_0) = (2 \sin b)^n \sin\{a + bx_0 + n(b + \pi)\}$ is true for $n = 1$ where $x_1 = x_0 + 2$.

18) In a society, number of members in some particular age interval is given as follows :

Age interval	$1-10$	$11-20$	$21-30$	$31-40$	$41-50$
Persons	20	15	8	19	12

19) Find number of members in the age interval $30 - 35$ using Gauss forward interpolation formula.

20) The function $y = \sin x$ is tabulated as $(0,0)$, $(\pi/4, 0.70711)$ and $(\pi/2, 1)$. Find the maximum possible error occurred in the interpolation of the given data points at $\pi/6$.

21) Taking necessary assumption derive the iterative formula for the method of false position. Also draw the diagram to represent it.

22) Evaluate the approximate square root of 8 up to two iterations correct up to four decimal places.

23) Tabulated values of some function $y = f(x)$ is:

x	1.1	1.2	1.3	1.4	1.5	1.6
y	-1.62628	0.15584	2.45256	5.39168	9.125001	3.83072

Find the slop of tangent of it at $x = 1.2$.

24) Using Runge-Kutta method of order four, find y at $x = 0.4$ given that $\frac{dy}{dx} = xy + x^2$, $y(0) = 1$ and $h = 0.2$.

25) Evaluate $\int_0^{\pi} \sqrt{\sin x}\, dx$ numerically taking 9 subintervals, using Simpson's method, correct up to four decimal places.

26) Find the approximate solution, using Gauss-Seidel's iterative method, of the system $3x + y + z = 2$,

27) $2x + y + 7z = 1$ and $x + 5y - 2z = 0$ up to second iteration correct up to 3 decimal places.

28) Show that $\Delta^n \sin(a+bx_0) = (2\sin b)^n \sin\{a+bx_0+n(b+\frac{\pi}{2})\}$ is true for $n=1$ where $x_1 = x_0 + 2$.

29) Express the polynomial $p(x) = 2x^3 - 3x^2 + 3x - 10$ into the factorial form.

30) Construct difference table up to second order of $y = \sin x$ for $0° \le x \le 30°$ with $\Delta x = 5°$.

31) State expression of the error estimation, occurring in the interpolation of n-number of data points.

32) Find the cubic polynomial which takes the following values; $y(1) = 24$, $y(3) = 120$, $y(5) = 336$ and $y(7) = 720$. Hence or otherwise obtain the values of $y(8)$.

33) Using Gauss's backward formula and find the sales for the year 1966, given that

Year	1931	1941	1951	1961	1971	1981
Sales In Lacs	12	15	20	23	39	52

34) Find the Lagrange's interpolating polynomial of degree 2 approximating the function $y = \ln x$ defined by the following values.

x	2	2.5	3.0
$\ln x$	0.69315	0.91629	1.09861

35) Interpolate the missing entries

x	0	1	2	3	4	5
y	0	----	8	15	----	15

36) Evaluate the coefficient of x^3 in the interpolating polynomial, generated by Newton's forward difference interpolation formula, satisfying $(-1, y_0)$, $(1, y_1)$, $(3, y_2)$ and $(5, y_3)$. Given that $\triangle^2 y_0 = 17$ and $\triangle^2 y_1 = 65$.

37) Estimate the maximum possible error, while interpolating the tabulated values (0,1), (0.5,0.6) and (1,0.3) of the function $y = e^{-x}$ at $x = 0.75$, using error expression for polynomial interpolation.

38) State the Gauss' forward and backward interpolation formula.

39) Apply Stirling's interpolation formula and find the most accurate population of a town in 1943, with the help of following data:

Year	1931	1941	1951	1961	1971
Population (Thousand)	15	20	27	39	52

40) Define the shift operator and using it evaluate the missing term in the following table.

X	0	1	2	3	4	5
y	0	----	8	15	----	35

41) Derive the inequality for finding the least number of iterations required to obtain the desired accuracy using Bisection method.

42) Derive the Lagrange's interpolation formula, by taking necessary assumption, for n number of data points.

43) Given that $(1,4)$, $(4,10)$,$(7,7)$ and $(10,22)$. Then for these tabulated values $f'(10) = 11$.

44) A root of the equation $10 \sin x = -x$ lies between 5 and 6.

45) Find interpolating polynomial passing through the points $(1,3)$, $(2, -1)$ and $(2.5, -3)$.

46) Find the minimum number of steps required to get the approximate root in the interval $(2,3)$ of the equation $x^3 - 4x - 9 = 0$ correct to three decimal places using bisection method.

47) Show that $\Delta^3 \equiv (E - 1)^3$, where Δ and E are the forward difference and shift operator respectively.

48) Use stirling's formula to find Y_{28}, given that $Y_{20} = 49225$, $Y_{25} = 48316$, $Y_{30} = 47236$, $Y_{35} = 45926$, $Y_{40} = 44306$.

49) Write general quadrature formula and derive the trapezoidal rule to evaluate $\int_a^b y(x)dx$ numerically.

50) Interpolate $f(110)$ corresponding to the data points $(40, 250)$, $(60, 370)$, $(80, 470)$, $(100, 540)$ and $(120, 590)$, using Newton's backward difference interpolation formula.

51) Use Newton-Raphson method to establish the iterative formula $x_{n+1} = \frac{1}{3}\left[2x_n + \frac{N}{x_n^2}\right]$ to calculate cube root of N.

52) Using Runge-Kutta method of order four, find y for $x = 0.6$, given that $\frac{dy}{dx} = xy + y^2$, $y(0) = 1$ and $h = 0.2$.

53) Write general quadrature formula and derive the Simpson's one-third rule to evaluate $\int_a^b y(x)dx$ numerically.